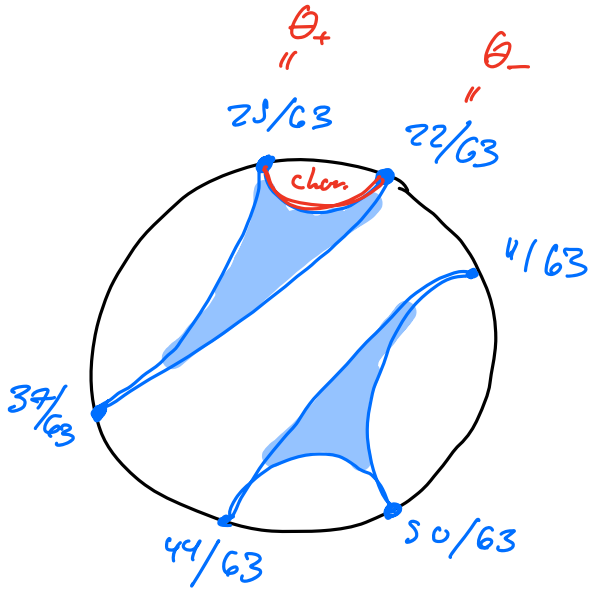


Rotational Parameter Rays

Ex: BTbFR

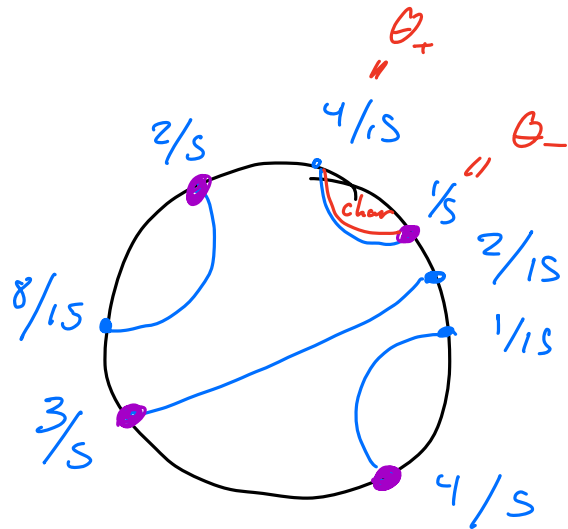
$$C = \frac{e^{2\pi i/3} - 1}{i}$$



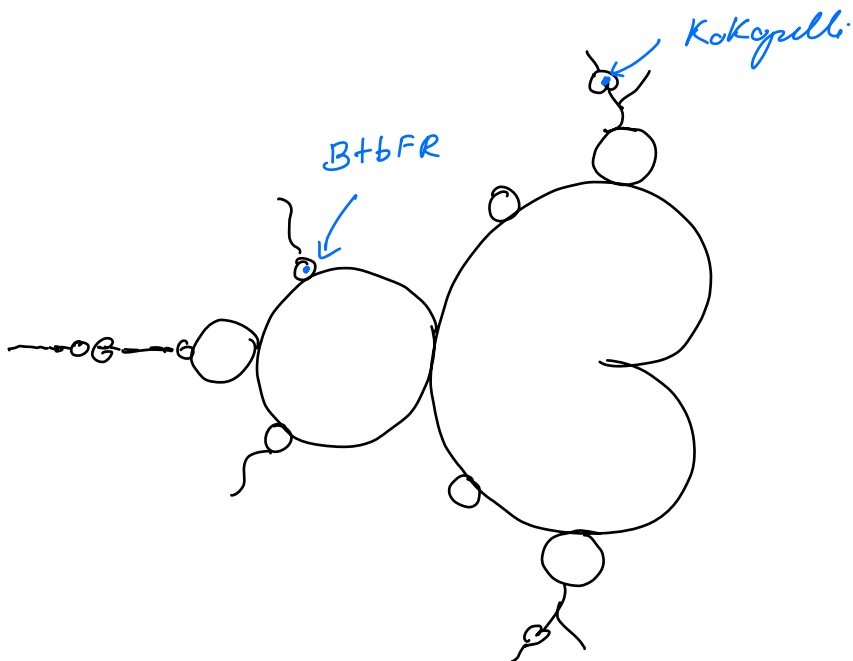
- Satellite
L 1 orbit under E

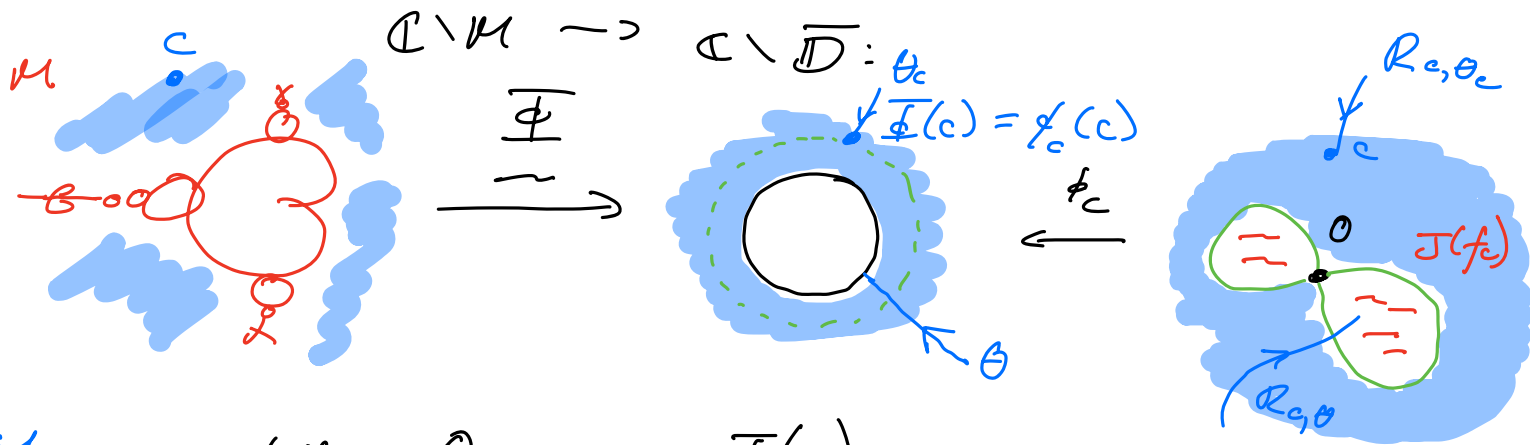
Ex: Kōkayelli

$$C = -0.15652 + 1.0322Si$$



- Primitive
+ 2 orbits under E





Lemma. $c \notin M$, $\theta_c = \arg \Phi(c)$.

1) $R_{c, \theta}$ lands at a point of $J(f_c)$ if

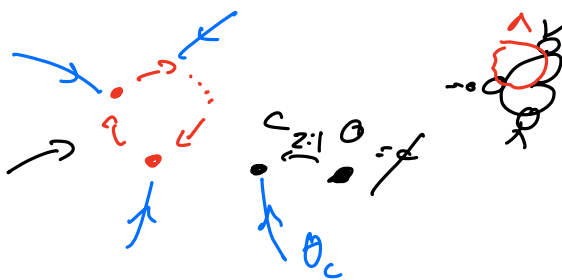
$E^n(\theta) \neq \theta_c$ for any $n \geq 1$.

not pre-critical

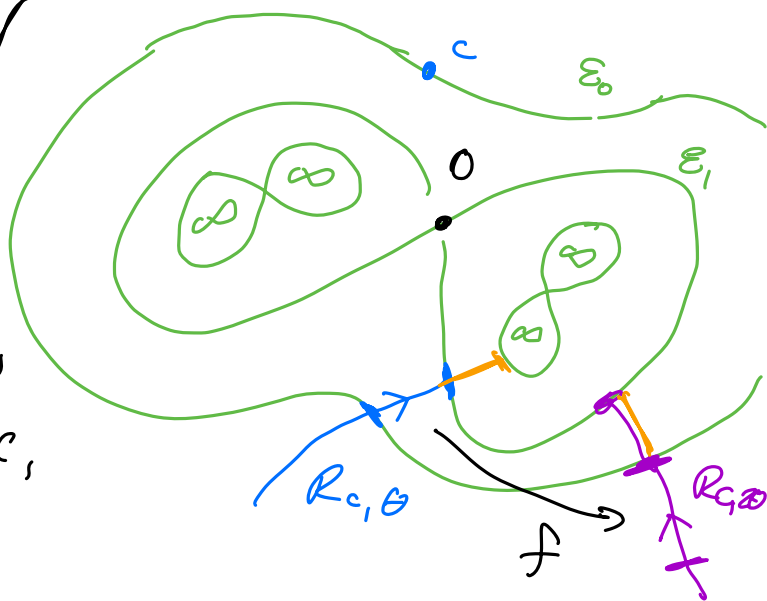


2) $\Lambda \subset \mathbb{C}$ open. If $E^n(\theta) \neq \theta_c$ for any $c \in \Lambda \cap M$, then the ray $R_{c, \theta}$ and its landing pt move continuously for $c \in \Lambda$.

Cor. Rays land on rep. cycles near hole in a neighborhood of c .

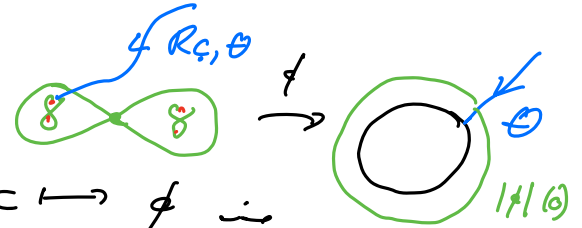


Proof. 1) $R_{c, \theta}$ is defined up to E_1 , and so is $R_{c, 2\theta}$. Since $E(\theta) \neq \theta_c$, $R_{c, 2\theta}$ does not land on c , so take a prime.



- 1.2) Galois part, size of $R_{c, 2\theta}$, goes to E_1 .
- 1.3) Repeat. Size of f is effectively, f^{-1} is constant, and hence $R_{c, \theta} \rightarrow \text{pt}$ of $\bar{D}$.
- 2) Under the assumption, for each $c \in \mathcal{A}$, $R_{c, \theta}$ levels in $\bar{D}(f_c)$.

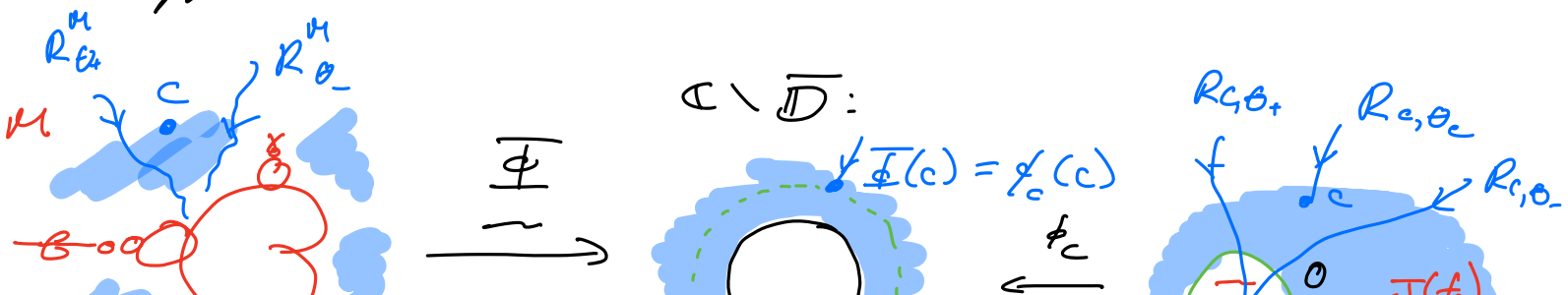
• Böttcher map is holomorphic $c: c \mapsto \phi_c$ is holomorphic, so $\phi_c^{-1} \{ r e^{2\pi i \theta} \mid r > 1/\psi(c) \}$ moves holomorphic as c varies. ψ_c solves the dynamical system, and by λ -lemma, the closure, including the landing pt.



Prop. A certain point, $\mathbb{I} = (\theta_-, \theta_+)$ char. are

1) $f_c(z) = z^2 + c$ has an orbiting point A iff $R_{\theta_-}, R_{\theta_+}$ land at the same point of $\bar{D}(f_c)$.

2) If $c \notin \mathcal{M}$, f_c has an orbiting point A iff $\theta_c \in \mathbb{I}$.

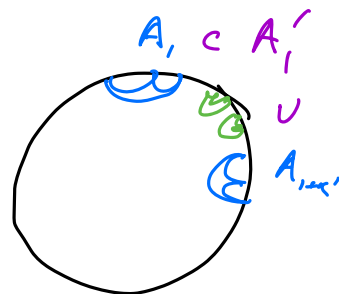


Cor. Any orbit partial is realized outside of M .

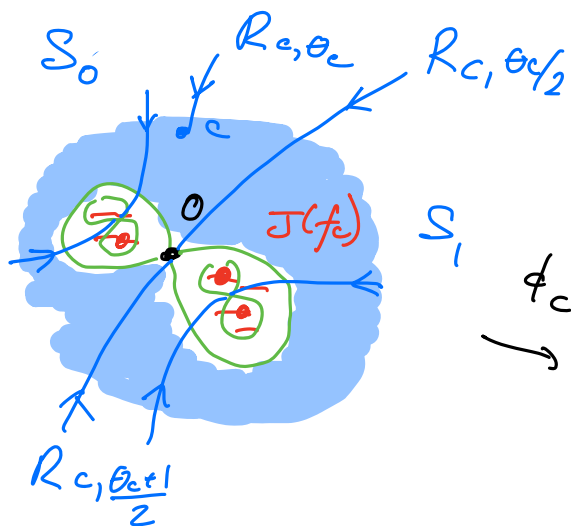
Proof. 1) $\Leftarrow$: $R_{\theta_{\pm}}$ land at $z_0 \in J(f_c)$, produce
 portait $A' = \{A_1', \dots, A_{k'}'\}$ for the orbit of z_0

1.1) A prim: $\theta_{\pm} \in A$, $\theta_{\pm} \in A_1'$ $\theta_{\pm}$ generate
 all of A , so $A_1 \subset A_1'$. $\theta_{\pm}$ are in sep. orbits $\Rightarrow A'$ is prim
 $\Rightarrow |A_1'| = |A_1| = 2 \Rightarrow A = A'$

1.2) A satellite: $\theta_{\pm} \in A$, are in the same
 orbit, generate $A' \Rightarrow A_1 \subset A_1'$
 if $k' < k$, then $A_{1+k'}' \subset A_1'$
 - Munking $\Rightarrow E$ can't take $A_{1+k'}'$ into A_1
 and preserve A_1'

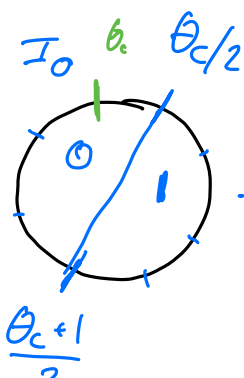


2) Recall: $c \notin M: J(f_c) \xrightarrow[\text{homeo}]{\sim} \underbrace{\{0, 1\}^{\mathbb{N}}}_{\text{Cantor set}}$



$z \mapsto \sigma(z)$

$\sigma(z)_i = \begin{cases} 0 & \text{if } f_c^i(z) \in S_0 \\ 1 & \text{if } f_c^i(z) \in S_1 \end{cases}$



For $\theta \in S'$, we
 let $\sigma(\theta)_i = \begin{cases} 0 & \text{if } E_i'(\theta) \neq \emptyset \\ 1 & \text{if } E_i'(\theta) = \emptyset \end{cases}$

R_{θ} land at $\sigma(\theta)$

2.1)

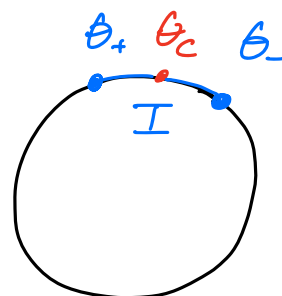
Suppose $\theta_c \in I$ ext. arc

Lemma.

Lemma.

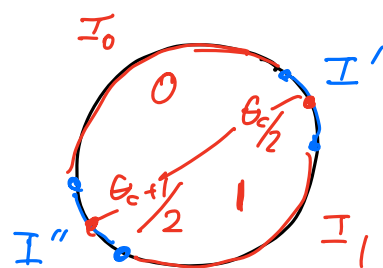
$$f^{-1}(I) = I' \cup I'', \quad \theta_{c/2} \in I', \quad \theta_{c+1/2} \in I'',$$

$$S' \setminus f^{-1}(I) = I_0 \sqcup I_1,$$



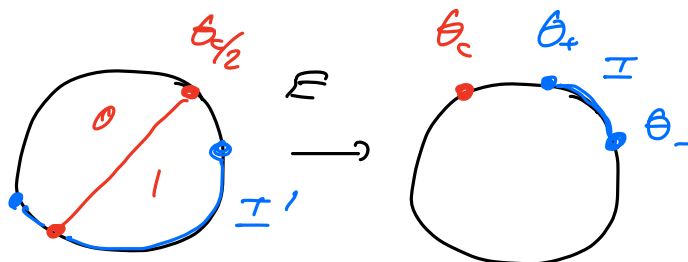
Monotonicity $\Rightarrow$ each A_i is either in I_0 or I_1 , $\uparrow \in$

$\Rightarrow$ all the angle in A_i have the same σ , land on the same point of J .



2.2) Suppose $\theta_c \in S' \setminus I$: I comp arc of A_i

$I = E(I')$ for I' a ext. arc of A_{i-1} .



Since $l(I') > \frac{1}{2}$, $\theta_c \notin I$, $\frac{\theta_c+1}{2}$ and I' covers all arc- I arcs once,

$\theta_{c/2}, (\theta_{c+1})/2$ separate the subarc of I' ,

so not all pts in A_{i-1} have the same σ ,

so they don't land in $J(f_c)$.

Def. $c \in \mathbb{C}$ is quasiregular if 0 is strictly preperiodic for $f_c(z) = z^2 + c$ Ex: $z^2 + i$

$\hookrightarrow$ locally conn. the orbit of 0 is strictly

permutative for E

$c \in \mathbb{C}$ is a permissible parameter of f_c is a permissible cycle Exclusive!

Thm. 1) If θ is permutative for E , then R_θ^{01} lands at a permissible parameter.

2) If $\theta \in \mathbb{Q}/\mathbb{Z}$, then R_θ^{01} lands at a point of M that is permissible or Mississippi parameter.

Proof. Lemma $\{c \text{ perm. or Mississ. param.}\}^{01}$ is stable:

Permissible: $f_c^n(z_0) = z_0, (f^n)'(z_0) = 1$

$0 = f_c^n(z) - z, 0 = \frac{d}{dz} (f^n(z) - z) \cdot \frac{\text{Discriminant of } f_c^n(z) - z}{f_c^n(z) - z}$

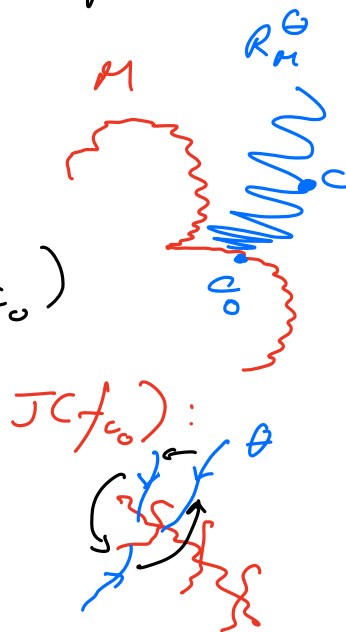
Mississ.: $f_c^n(0) = f^n(0)$

c_0 be an accumulation pt. of R_θ^{01}

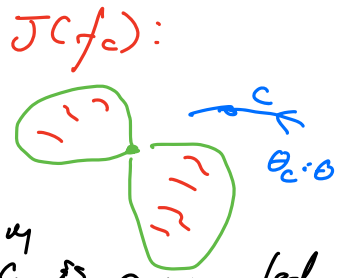
1) $E^n(\theta) = \theta, R_{c_0, \theta}$ lands on $J(f_{c_0})$ at a point that is repelling or permissible.

Repelling $\Rightarrow R_{c, \theta}, R_{c, E(\theta)}, \dots$

all land + none land in C near c_0



But for $c \in R_\Theta^u$, $\Theta_c = \Theta$, and
 $E^u(\Theta) = \Theta_c$, so the rays can't land.
 $\Rightarrow C_0$ is generic point. Accumulation pts of R_Θ^u is connected.

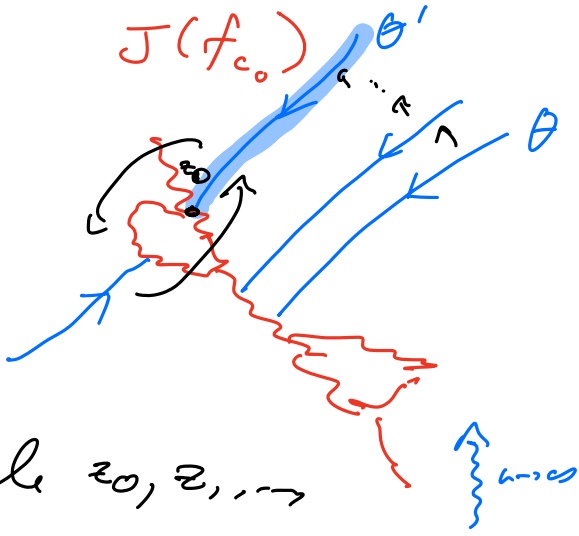


2) $E^u(\Theta) = \Theta'$, $E^u(\Theta') = \Theta'$

c_0 accum. pt of R_Θ^u . Suppose
 c_0 is not generic

$\Rightarrow R_{c_0, \Theta'}, \dots, R_{c_0, E^{n-1}(\Theta')}$ all

land on a regelling cycle $z_0, z_1, \dots$
 moves holomorphically w/ c .

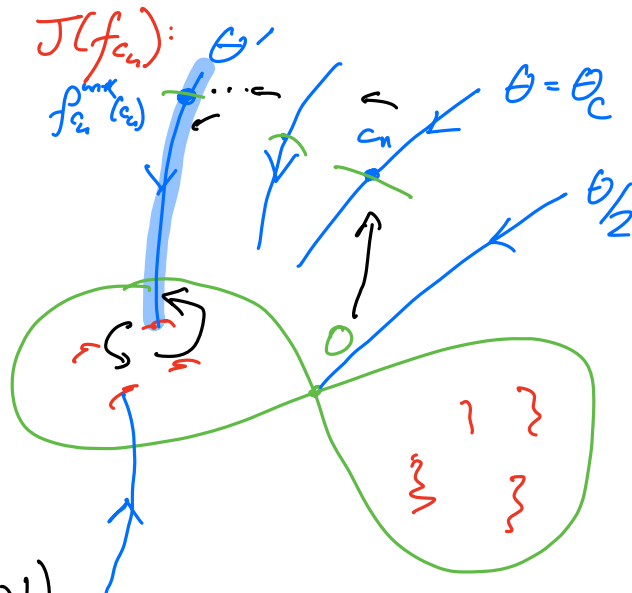


$R_{c, \Theta}(t) = f_c^{-1}(e^{2\pi i \Theta t})$

Take $c_n \in R_\Theta^u$, $c_n \rightarrow c_0$

$f_{c_0}^{u+1}(0) = \lim_{u \rightarrow \infty} f_{c_n}^{u+1}(0)$

$= \lim_{u \rightarrow \infty} R_{c_n, \Theta'}(2^u \underbrace{\log |f(c_n)|}_{\rightarrow 0})$



C_0 is Misirrewing

$= z_0, \dots$
 Holomorphic
 of rays,
 λ -lemma
 for count

Set of Misirrewing is countable.